

## Geometrical Optical Illusions and Peirce's "Fourth" Cotary Proposition

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**Abstract:** After delivering the seventh of his Harvard *Lectures on Pragmatism*, Peirce added a passage to the manuscript stating, but not defending, what I call his fourth cotary proposition: The process that results in a perceptual judgment, and the steps that constitute that process, if subjected to logical analysis, would all be found to have the form of an abductive inference. This paper argues that the contemporary research of cognitive scientists into geometrical optical illusions supports Peirce's claim.

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After delivering the seventh of his Harvard *Lectures on Pragmatism*, in which he presents his three cotary propositions, Peirce added to the manuscript what I shall call his *fourth* cotary proposition: The process that results in a perceptual judgment, and the steps that constitute that process, if subjected to logical analysis, would all be found to have the form of an abductive inference.<sup>1</sup> Here are Peirce's own words:

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<sup>1</sup> What I call Peirce's fourth cotary proposition is importantly different from the preceding three. The three cotary propositions all concern perceptual judgment and intellection (be it conception or inference). They are:

- (1) There is nothing in the intellect that was not previously in the senses or, as and by "in the senses" he specifically means perceptual judgment (EP 2.226)
- (2) "Perceptual judgments contain general elements" (EP2.227).

On its side, the perceptive judgment is the result of a process, although of a process not sufficiently conscious to be controlled, or, to state it more truly, not controllable and therefore not fully conscious. If we were to subject this subconscious process to logical analysis, we should find that it terminated in what that analysis would represent as an abductive inference, resting on the result of a similar process which a similar logical analysis would represent to be terminated by a similar abductive inference, and so on *ad infinitum* (EP 2.227)

Consider, for example, the process when I see a red cube and judge “That is red.”

We may schematize it as follows:

I. (visual sensorimotor coupling with the cube + intervening uncontrolled, unconscious process) → “That is red”

Here, → symbolizes what the visual sensorimotor coupling with the cube and the intervening unconscious cognitive process yields (here, a perceptual judgment). Peirce’s claim is that *if we treat the intervening uncontrolled process as having an inference form* then we should find that it has the following form, which is an abductive inference:

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(3) “Abductive inference shades into perceptual judgment without any sharp line of demarcation between them” (EP2.227).

The whole weight of the seventh lecture consists in proving (3), for Peirce thinks that if it is proven then the first two cotary propositions follow.

In contrast with the first three, the fourth cotary proposition concerns that unconscious, uncontrollable process whereby perceptual judgments themselves are formed.

- (i) Light stimulates my visual system in such-and-such a way (visual sensorimotor coupling).
- (ii) If the cube were red, then it would be not be surprising that my visual system were so stimulated.
- (iii) Therefore, I have reason to suspect that the cube is red.

Peirce's fourth cotary proposition is quite far-reaching in at least three different respects. First, it is not restricted to any particular sense modality. Hence, even this more general schema:

**II.** (sensorimotor coupling + intervening uncontrolled process) → perceptual judgment

Would have the form an abductive inference if subjected to logical analysis. Second, Peirce indicates that if we divide II into parts, we shall find that the termination of each part has the form of an abductive inference. He grants that between sensory inputs and perceptual judgments are percepts.<sup>2</sup> Thus, we should find that both:

**IIa.** (sensorimotor coupling + intervening uncontrolled process) → percept

And:

**IIb.** (percept + intervening uncontrolled process) → perceptual judgment

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<sup>2</sup> See, for example, CP 7.628: "The perceptual judgment professes to represent the percept".

have the forms of abductive inference. Third, Peirce claims that this would be true if we divided the processes *ad infinitum*.

As a later addition to the manuscript, Peirce's fourth cotary proposition is nowhere defended in the seventh lecture. Indeed, Peirce's claim is very strong and, perhaps, implausible.<sup>3</sup> Nonetheless, what I shall contend here is that contemporary research into geometrical optical illusions lends *partial* support to Peirce's fourth cotary proposition.<sup>4</sup> In particular, the research supports the claim that the process involved in:

**III.** (visual sensorimotor coupling + intervening uncontrolled process) → visual percept

has the form of an abductive inference.<sup>5</sup> First, I discuss the nature of abductive inference; Second, I canvass some of the contemporary research into geometrical optical illusions and argue that the process of III has the form of an abductive inference; Third, I consider some objections and reply.

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<sup>3</sup> One point of worry is Peirce's claim that even if the process were analyzed *ad infinitum*, every step would be found to have the form of an abductive inference and that this is true of *every* sense modality. A second cause for worry arises from the complexity of analyzing percepts that integrate perceptual modalities (for example, perceiving a bouncing, sounding ball, which integrates both visual and aural perceptions).

<sup>4</sup> I do not think research into geometrical optical illusions alone supports Peirce's fourth cotary proposition. Research into other sorts of illusions, hallucinations, cognitive penetration, and synesthesia all lend support to the claim that IIIa has the form of an abductive inference. However, space prevents me from detailing those features of the argument here.

<sup>5</sup> I shall not here address what reasons Peirce himself had to endorse the fourth cotary proposition; rather, my concern is with whether contemporary research—here limited to research into geometrical optical illusions—supports it.

## The Form of Abductive Inference

Importantly, Peirce's claim is that the process involved in III has the *form* of an abductive inference (or, as Peirce puts it, that logical analysis would "represent" it as an abductive inference). It is not in fact an abductive inference because on Peirce's view all inference is self-controlled reasoning (see EP 2.463). However, as he states, III is not self-controlled; it is not even controllable.<sup>6</sup>

What is the form of an abductive inference? Peirce's canonical statement is given in the very same lecture as the fourth cotary proposition:

The surprising fact, C, is observed;  
But if A were true, C would be a matter of course,  
Hence, there is reason to suspect that A is true (CP 5.189, 1903).

In two excellent recent essays written on Peirce's theory of abductive inference, both Douglas Niño (2009) and Daniel Campos (2009) have argued that abductive inference is not the same as inference to the best explanation, which is widely regarded as a kind of induction, at least on the accounts of Peter Lipton and Gilbert Harman.

Although Niño and Campos do not state it in quite this way, we can get to the heart of the issue by distinguishing between two senses of "inference to the best explanation." The first sense is inference *as a guess at* the best explanation. The second is inference *as a discernment of* the best explanation. Inference as a guess at the best

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<sup>6</sup> This is an important, but little noted, change from Peirce's position in 1868's "Questions Concerning Certain Faculties Claimed for Man," where Peirce claims not just that it has the form of an inference but it is an inference (see W 2.196; see also 2.235).

explanation merely proposes a plausible hypothesis to tentatively adopt. This is abduction. Inference as a discernment of the best explanation concludes that some hypothesis is probably true because it predicts the phenomena better than any competing hypothesis. This is a species of induction. Campos argues that Lipton's theory involves a conflation of these two sorts of inference, a conflation "that blurs an important distinction between the tentative, exploratory inference that first suggests a plausible explanatory hypothesis and the probabilistic inductive inference that admits or rejects a hypothesis as scientifically tenable" (440–441).

Following Niño and Campos, three points are especially helpful for getting a handle on what form abductive inferences take, especially in contrast with inductive inferences.

(1) Abductions, like all forms of inference, have some premises and a conclusion. The conclusion interprets the premises.

(2) Abduction provides us with hypotheses to test whereas induction tests those hypotheses. For example, it is a surprising fact that I have never seen a non-white swan. If all swans were white, that surprising fact would be a matter of course. Now, I have a reason to suspect that all swans are white; that is the hypothesis I shall test. To do so, I perform an induction. I predesignate that I shall study swans with respect their whiteness and then I set about sampling swans.

(3) Abductive inferences do not merit the assertion of—they do not assume responsibility for the truth of (see CP 5.546)—their conclusions (see EP 2.287). They only "suggest that something [the conclusion] may be" (EP 2.216). In contrast, induction yields conclusions that are probable and assertible.

Peirce's claim is that when the intervening process of III is treated as an "inference," we should find it has the following form:

- (i) Light stimulates my visual system in such-and-such a way.
- (ii) If the object were such-and-such a way (the percept), then it would not be surprising that light stimulates my visual system in such-and-such a way.
- (iii) Therefore, the object may be such-and-such a way (the percept).<sup>7</sup>

That is to say: (1) the percept interprets the input from sensorimotor coupling; (2) the percept provides us with a hypothesis of the way the world is that may be tested; but (3) the percept does not, in and of itself, assert that the world is in fact the way it is presented in the percept.

### **Contemporary Research into Geometrical Optical Illusions**

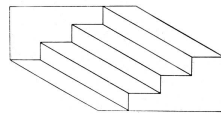
Research into geometrical optical illusions shows that visual percepts are guesses or fallible insights into the way the world is, and so such research supports the claim that the process of III, if subjected to logical analysis, would be found to have the form of an abductive inference. Such illusions reveal this because they are instances when the visual system fails to guess correctly, when its fallibility comes to the fore. In particular, they

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<sup>7</sup> There are, obviously, grammatical challenges in representing III as an argument, since a percept is not a sentence or any other sort of grammatical representation. But that supports what was said before: III is not an inference but can be represented as having the form of an abductive inference. The sense in which it has the form of an abductive inference is not the sense of it having the grammatical form of abductive inference but the characteristics of an abductive inference, as in points (1)–(3).

are instances when the visual system fails to guess correctly either because (a) there are insufficient visual cues; (b) the visual system itself is limited; or (c) the subject is not in forward motion. After examining why each of these scenarios results in an illusion, I shall argue that they show III has the form of an abductive inference.

*Insufficient Visual Cues:* The traditional inference theory is that illusions are a consequence of the visual system “inferring” how the world in fact is. This theory has its origins in the work of Helmholtz, and it surely the theory with which Peirce would have been familiar. According to the traditional inference theory, some visual illusions are a consequence of the perceiver having insufficient visual cues as to how the world is. To illustrate why such illusions arise, we can employ an example Peirce used,<sup>8</sup> Schroeder’s Stair:



Note that this figure may be view either as ascending from left to right (a staircase) or as suspended and descending from right to left (a cornice). According to the traditional

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<sup>8</sup> Peirce uses the example to support his second cotary proposition that perceptual judgments contain general elements, but he also states that the “percept itself seem[s] to keep shifting from one general aspect to the other and back again” (EP 2.228).



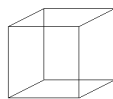
inference theory, the lack of visual cues makes it such that the visual stimuli may be interpreted by the visual system in either way.<sup>9,10</sup>

*Limitations of the Visual System:* Cornelia Fermüller, with others, has argued that geometrical optical illusions are also caused by limitations of the visual system itself. The early visual system extracts measurements as to the value intensity, edges, and motion of images. However, it can do so with only limited accuracy. Its accuracy is limited for a variety of reasons, among them blurring caused by the lenses of the eyes, the movement of the eyes themselves, and images received at different times needing to be combined. Whereas those limitations are insignificant for most practical affairs, they become exaggerated with respect to geometrical optical illusions.

One example of such an illusion is the café tiles illusion:

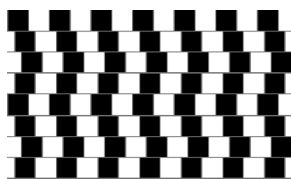
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<sup>9</sup> Note that the question here is not simply one of conceptual application, of perceiving as. To see why, consider another reversible image, the cube, with respect to which we really can switch between seeing one face forward and another behind:



Letting the two left-most vertices be *ab*, the next two *ef*, the next two *cd*, and the two right-most *gh*: we do not simply see the face *abcd* *as forward* but actually see *the forward face abcd*. This is true also if we switch to seeing the forward face *efgh*. We do not merely see the face *efgh* *as forward*.

<sup>10</sup> What visual cues are missing from Schroeder's Stair? In all likelihood, there is no singular visual cue that is missing. Rather, the figure might be disambiguated if luminance contrasts were added to it (see Williams, McCoy, and Purves, 1998). It might also be disambiguated if more information were added to the scene, such as a person ascending the staircase (as is done in the works of MC Escher, for example). Nonetheless, the key point is the same: the illusion occurs because the visual system does not have sufficient information to guess correctly as to how the staircase is in fact oriented.



The thin horizontal lines of medium luminance (the mortar) of this image are parallel even though they do not appear so. On the present account of illusions, this occurs because the visual system tries to smooth the two edges of the black and white tiles (of low and high luminance respectively) into one edge, thus drawing the edges of the black and white tiles over the mortar, so to speak, and distorting the line.

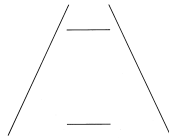
*Forward Motion:* The Perceiving-the-Present theory of visual illusions is a refinement of the traditional inference theory.<sup>11</sup> According to it, the visual system does not merely “infer” how the world *is* but how the world *will be*. The perceiving-the-present theory of visual illusions developed by Mark Changizi is that some visual illusions are a consequence of the visual system compensating for neural delay involved in processing visual information. 100 milliseconds elapse between the time a visual stimulus is received and a visual percept occurs. However, if one is moving forward at a rate of approximately 2 miles per hour (1 meter per second) when visually perceiving, that means one’s own body in relation to one’s surroundings have changed approximately 10 centimeters during that time. Thus, the visual system must compensate for that neural

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<sup>11</sup> See 2001, though in his later he work is coyer about this fact.

delay in order to avoid possible harms (e.g. collision) or gain benefits (e.g. capturing prey).<sup>12</sup>

Changizi et. al. (2008) have argued that this theory explains a number of illusions, including illusions of size, speed, luminance, and distance. As one example, it explains the Ponzo illusion:



The more distant line in the perceiver's presumed direction of motion (straight ahead) appears longer because of its expected expansion as the perceiver moves forward along the path of sloping vertical lines.<sup>13</sup>

*Geometrical Optical Illusions and Abduction:* According to the inference theory, geometrical optical illusions are the result of failures of the visual system to correctly “infer” the way the world is or the way it will be. But why think that the “inference” in question is an abductive inference rather than a deductive or inductive inference?

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<sup>12</sup> Moreover, as the costs are less severe for latency compensation to be elicited when *not* in motion than for latency compensation to *fail* to be elicited when in motion, some visual illusions are simply by-products of latency compensation being made when not in motion, as evolutionary processes would select for visual systems that do make such compensations (obviously, since one is less likely to suffer harms (such as collision) and more like to gain benefits (such a captured prey)).

<sup>13</sup> I should note that Fermüller and Changizi take themselves to have explained some of the same illusions. Ascertaining whose explanation is correct is not a concern of this paper. Both theories support the claim that III has the form an abductive inference.

We begin by assuming that the process can be represented to have some inference form. Such an assumption is fundamental to the cognitive sciences and computational, operational, and functionalist theories of mind. It is also plausible to treat the process as an inference insofar as the percept is akin to an interpretation of the sensorimotor input, much as the conclusion of an inference is an interpretation of the premises.

Next, assume with Peirce that there are only three possible argument forms: deduction, induction, and abduction. However, the percept cannot be an interpretation of the visual input in the same way that the conclusions of deductions are interpretations of their premises. For the percept is not necessitated by the visual input itself. We can see the staircase or the cornice (though not both at once). However, in each case, the sensorimotor input is identical.<sup>14</sup> In like manner, there is no reason to think the visual system *must* see the mortar lines of the café tiles illusion as non-parallel or the upper horizontal line as longer in the Ponzo illusion. Indeed, there is some indication that persons raised in non-carpenentered environments are less prone to certain of the geometrical optical illusions.<sup>15</sup>

Perhaps, then, the inference is inductive. As the work of Campos and Niño has shown, however, it is important to disentangle “inference to the best explanation” as inductive and as abductive. Granting that (1) the percept is an interpretation of the visual input, the question thus becomes whether percepts (2) test hypotheses and (3) warrant their own assertion, so to speak. If the answer is affirmative, then the process should be

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<sup>14</sup> Keep in mind that the intervening uncontrollable processes just is what we are representing as an inference form.

<sup>15</sup> See Gregory (1997), 150–51.

represented as an induction. If it does not involve (2) and (3), then it must be an abduction.

However, it is clear that (2) the percept does not test hypotheses and (3) the percept does not *alone* merit the assertion that the world is in fact the way it is presented. As to (2), there is no hypothesis being tested when we see the staircase. Rather, we would test it if we decided to see whether something could ascend the staircase, for example. Seeing the staircase is something that happens to us; testing is something that we do.<sup>16</sup> This is especially evident if we consider the other two illusions. We have no control over whether we see the mortar lines as parallel or not. Also, we have no control over whether we see the upper horizontal line as longer or not. It is only after suffering the illusion that we may test whether the mortar lines are indeed parallel or whether the upper line of the Ponzo illusion is indeed longer.

As to (3), Schroeder's Stair does not warrant the assertion that it is a staircase for it could as easily be asserted it is a cornice. The same is true of the café tiles illusion and the Ponzo illusion. I know that the café tile mortar lines are parallel, and yet I can do little to alter the illusion. Similarly, I know that the two lines of the Ponzo illusion are equal, but I can do nothing to make them appear so.

It follows that the process that results in a percept must not be inductive in form. As we have seen, neither is it deductive. Granting that there are only three inference forms, it follows that the process must have the form of an abductive inference, if it can be represented to have any inference form at all.

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<sup>16</sup> Of course, we might test whether and how we are able flip between seeing the stairs or the cornice, but then we are testing something about ourselves and not about the figure.

## Objections and Replies

We turn now to three lines of objection against the above line of reasoning. The first objection is that the visual system guesses right too often for us to regard the process that results in percepts as having the form of an abductive inference.

But this objection is a mistake. There is nothing intrinsic to abduction that requires our guess to be wrong with any high degree of frequency. Indeed, the visual system's capacity to guess rightly so often is why Peirce states that "the abductive faculty...is...a gradation of that which in its highest perfection we call perception" (EP 2.224).

A second line of objection: contemporary research into visual illusions is modeled on Bayesian probability theory. So, it would seem that what our visual system is doing is inferring the way the world is based on past experience, not making guesses.

But this objection is mistaken in two ways. First, it conflates the way in which we model the visual system with what the visual system actually does. To use an analogy, even if we can model human thought using computer programs, it does not follow that thinking just is to run the brain's computer program, so to speak. In like manner, just because we can model the processes of the visual system using Bayesian probability theory, it does not follow that the visual system actually does draw Bayesian inferences.

Second, the appeal to past experience is irrelevant to whether the process of the visual system is abductive or inductive in form. For starters, past experiences can certainly inform our abductions; indeed, it would be hard to imagine how anyone could draw abductive inferences without appealing to past experiences. Second, what does

matter is whether the visual system tests hypotheses, thereby engaging in predesignation and sampling, and whether percepts warrant the assertion that the world is the way it is presented. The answers, as argued above, are no.

A third objection comes along the lines of the theory of evolution. For it might be supposed that the appropriate argument form to represent the process in III is inductive insofar as evolution has selected for perceptual systems that perceive the world as it is or will be. Thus, there is a sense in which it might be said evolutionary processes have “tested” our visual systems and “asserted” (by way of natural selection) that only the ones that veridically present the world shall survive.

However, even if we grant this line of argument, it does not show that III does not have the form of an abductive inference. It only shows that evolution has favored visual systems that guess rightly about the world most of the time. In other words, the argument does not tell us anything about the inference form of III. It only tells us something about the inference forms of evolutionary processes. For his part, Peirce certainly thought that nature—in some abused sense of the term—“reasons.”<sup>17</sup> And so he would allow that nature tests its own hypotheses, in a sense. But, again, it does not follow from this that the process the visual system itself employs has the form of an induction.

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